WiOpt, Paris, France

Distributed Power Control and Coded Power Control

S. Lasaulce(*), joint work with [authors]
(*) L2S (CNRS–CentraleSupelec)
lasaulce@lss.supelec.fr

May 18, 2017

Introduction

What is meant by ”distributed” power control

- **Distributed decision-wise and information-wise**

$$u_i(p_1, \ldots, p_i, \ldots, p_K; G)$$

What is meant by ”distributed” power control

► **Distributed decision-wise and information-wise**

partial control

$$u_i(p_1, \ldots, \boxed{p_i}, \ldots, p_K; G)$$

What is meant by "distributed" power control

► **Distributed decision-wise and information-wise**

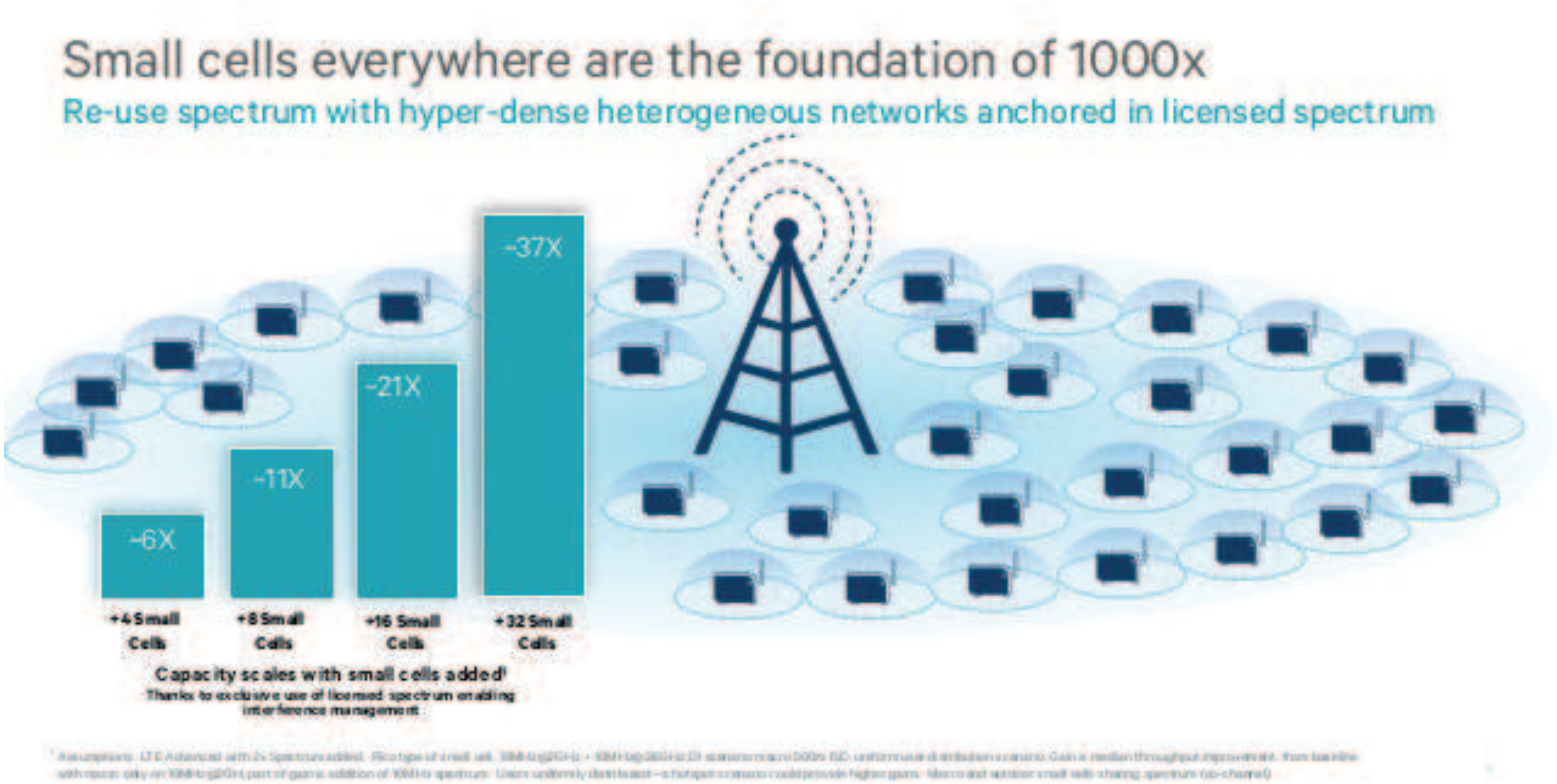
partial control

$$u_i(p_1, \ldots, \boxed{p_i}, \ldots, p_K; G)$$

$$s_i \text{ or } y_i$$

partial observation

Motivation for distributed power control



Distributed power control algorithms: Typical conclusion

- Local decision
- Local information
- (Affordable complexity)
- Global inefficiency

Natural questions

Distributedness & global efficiency :
Unmarriable features?

What is the best we can do with what know?

Take away messages

- **Power control strategy = code**

- **Limiting performance of power control $\rightarrow$ information theory**

Outline

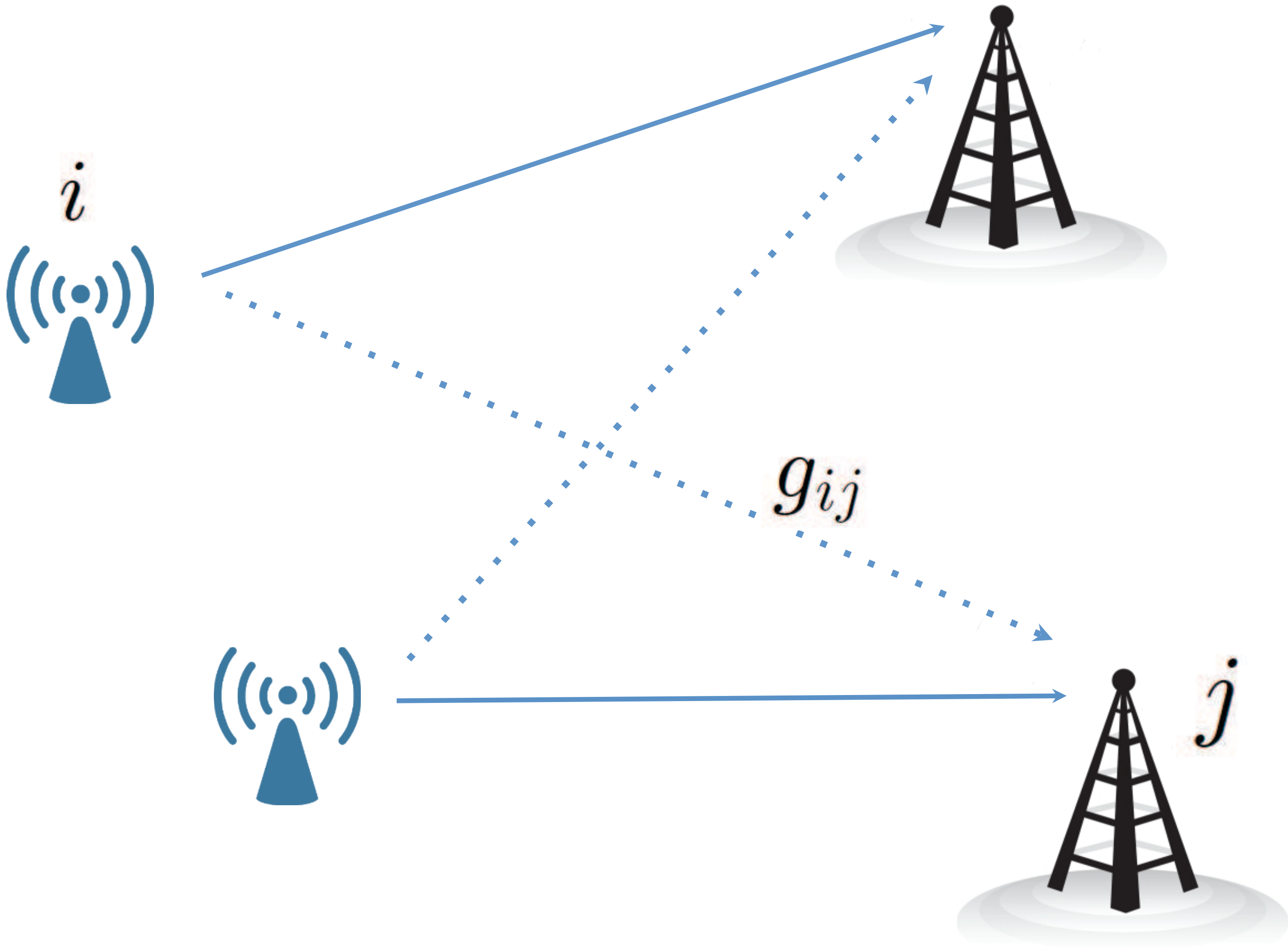
- Power modulation
- Limiting performance of coded power control
- Power control code example

Power Modulation

▶ Performance measure:

$$u(p_1, ..., p_K; g_{11}, ..., g_{KK}) = u(p_1, ..., p_K; \boldsymbol{G})$$

Considered interference network (Here $K = 2$, $S = 1$)



Global inefficiency: What is the problem?

- **Assumed utility form:**

$$u(p_1, p_2; g_{11}, g_{12}, g_{21}, g_{22}) = u(p_1, p_2; \boldsymbol{G})$$

- **Classical example:**

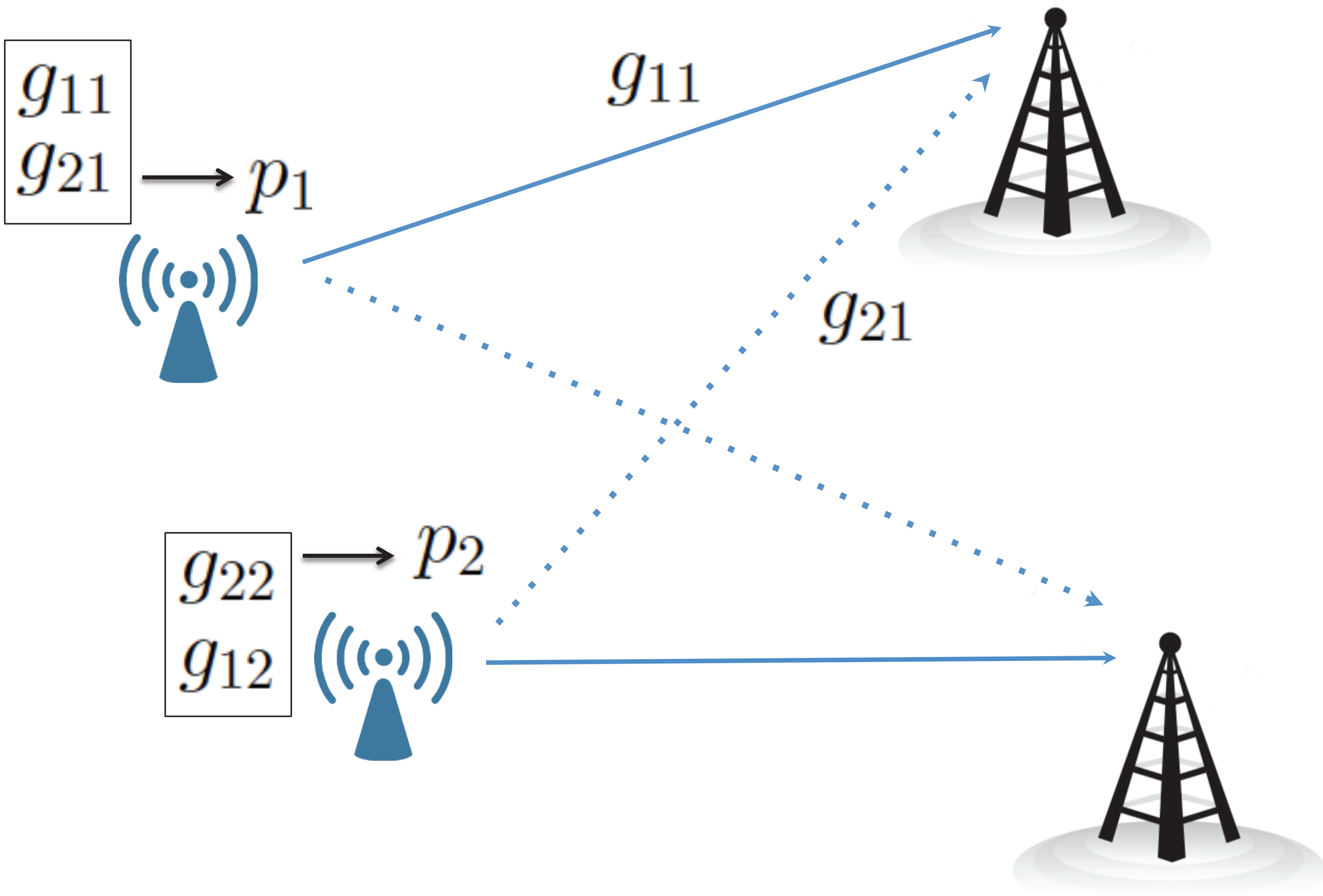
$$u_{\text{sum-rate}}(p_1, p_2; \boldsymbol{G}) = \sum_{i=1}^{2} \log\left(1 + \text{SINR}_i\right)$$

Global inefficiency: What is the problem?

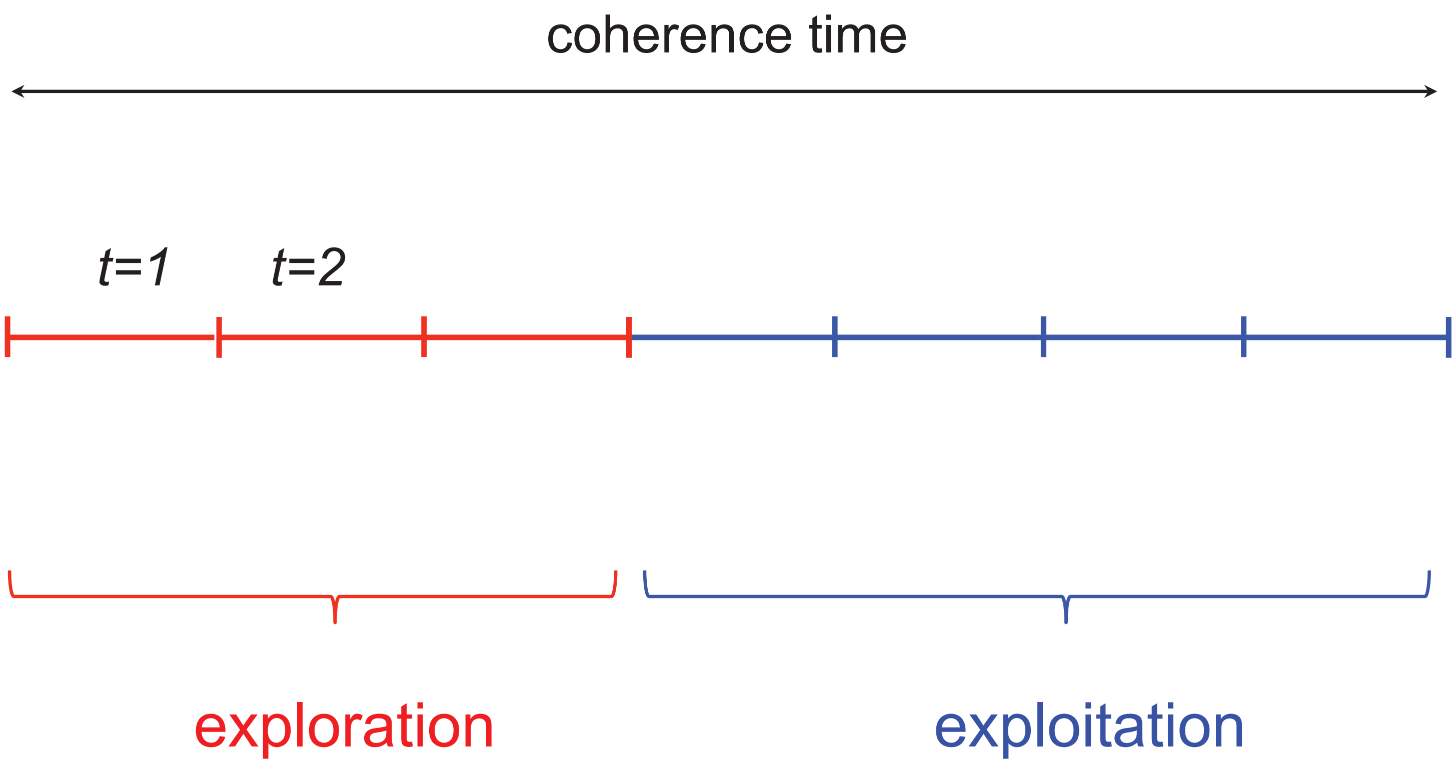
- **Complexity issue:** maximizing u may be hard

- **Information availability:** global CSI G typically not available at the Tx. How to solve this issue?

The power modulation idea [Varma 2015][Zhang 2017]



Two main phases



How to find p_1 at Tx 2?

- **OK with SINR feedback**

$$\mathrm{SINR}_2 = \frac{g_{22}p_2}{\sigma^2 + g_{12}p_1}$$

- **OK with RSP feedback**

$$\omega_2 = \sigma^2 + g_{22}p_2 + g_{12}p_1$$

Local CSI estimation

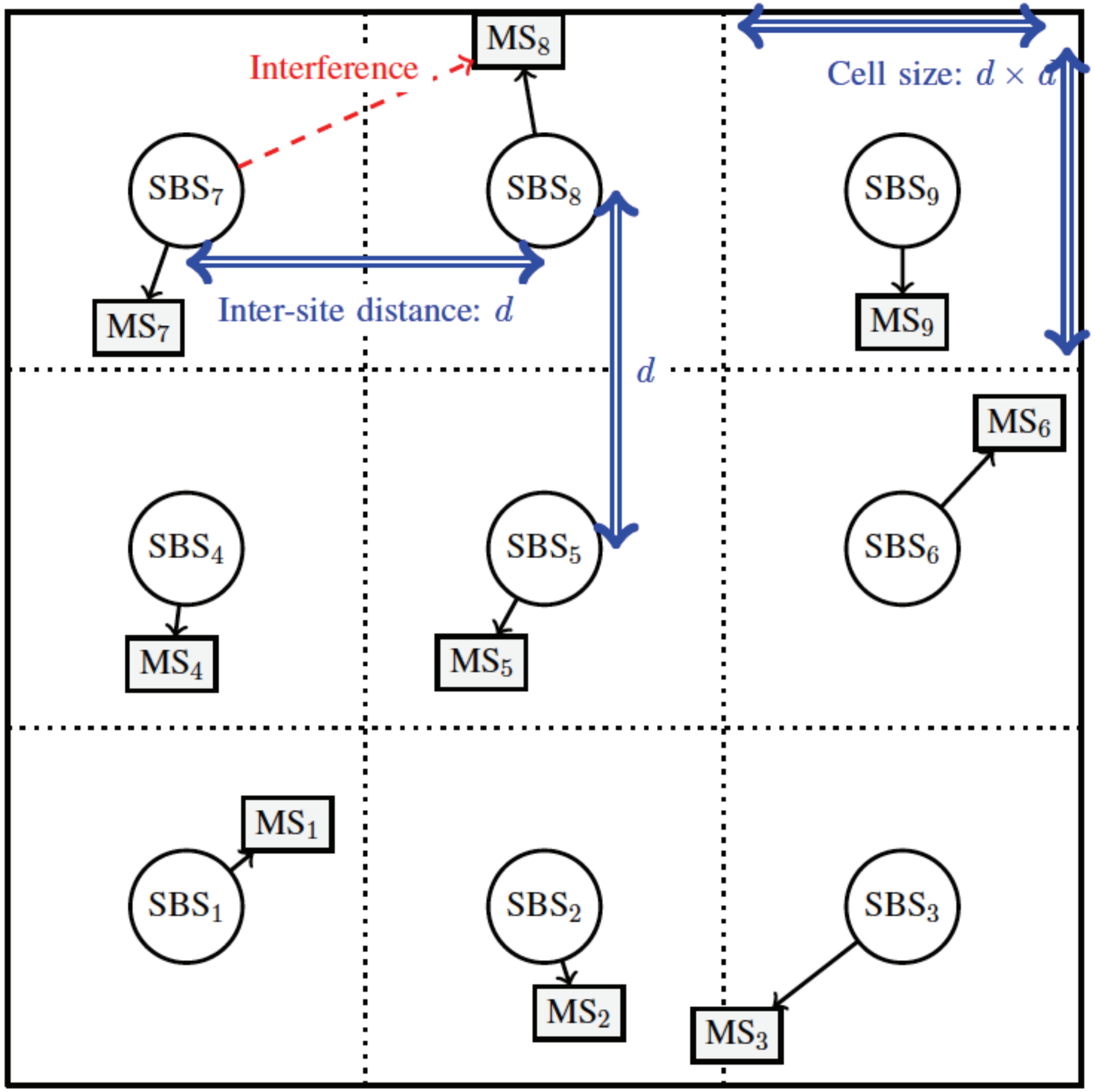
- **Training matrix:**

$$\mathbf{P} = \begin{pmatrix} p_1(1) & p_2(1) \\ \vdots & \vdots \\ p_1(N) & p_2(N) \end{pmatrix}.$$

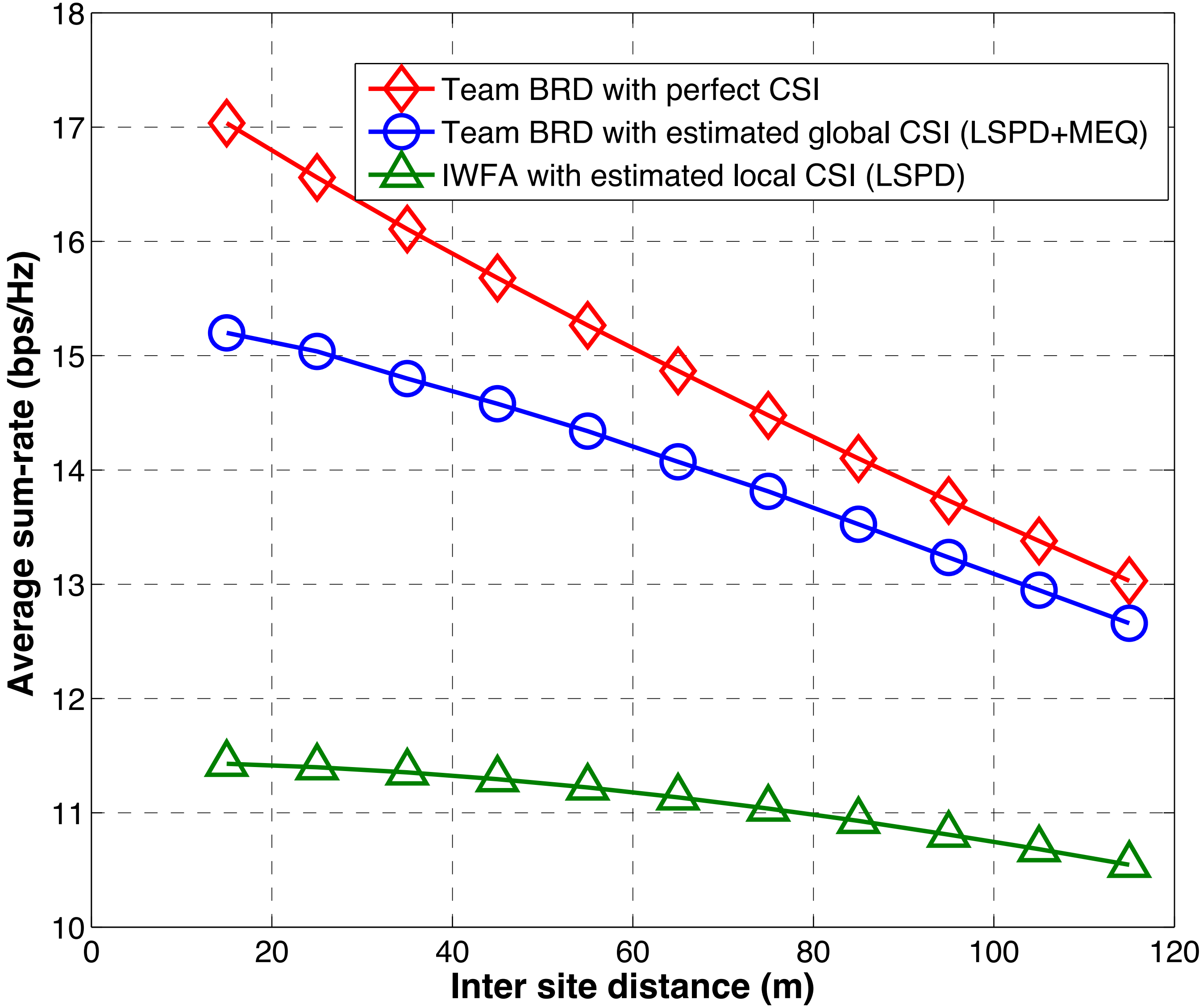
- **Power domain observation equation:**

$$\begin{pmatrix} \omega_1(1) \\ \vdots \\ \omega_1(N) \end{pmatrix} = \mathbf{P} \times \begin{pmatrix} g_{11} \\ g_{21} \end{pmatrix} + \sigma^2 \underline{1}$$

Numerical analysis: Scenario



Numerical analysis: Simulations



Take away messages

- ► RSP/SINR feedback is sufficient to reconstruct global CSI. Maximal efficiency can be theoretically achieved.

- ► Key ingredient: RSP/SINR = communication channel + power modulation. Use interference to manage interference.

- ► Other types of (structured) feedbacks, other types of exchange information, ...

Limiting Performance of Coded Power Control

Available global CSI image: $\Gamma_i(s_i|g_{11}, ..., g_{KK})$, everything discrete

Special cases

- Global CSI: $s_i = (g_{11}, g_{12}, ..., g_{KK})$
- Individual CSI: $s_i = g_{ii}$
- Imperfect individual CSI: $s_i = \widehat{g}_{ii}$

Limiting performance of power control

Power control strategy (causal case):

$$f_{i,t} : (s_i(1), ..., s_i(t)) \mapsto p_i(t)$$

Utility:

$$u_i^\infty(f_1, ..., f_K) = \lim_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left[u_i(p_1(t), ..., p_K(t); g(t))\right]$$

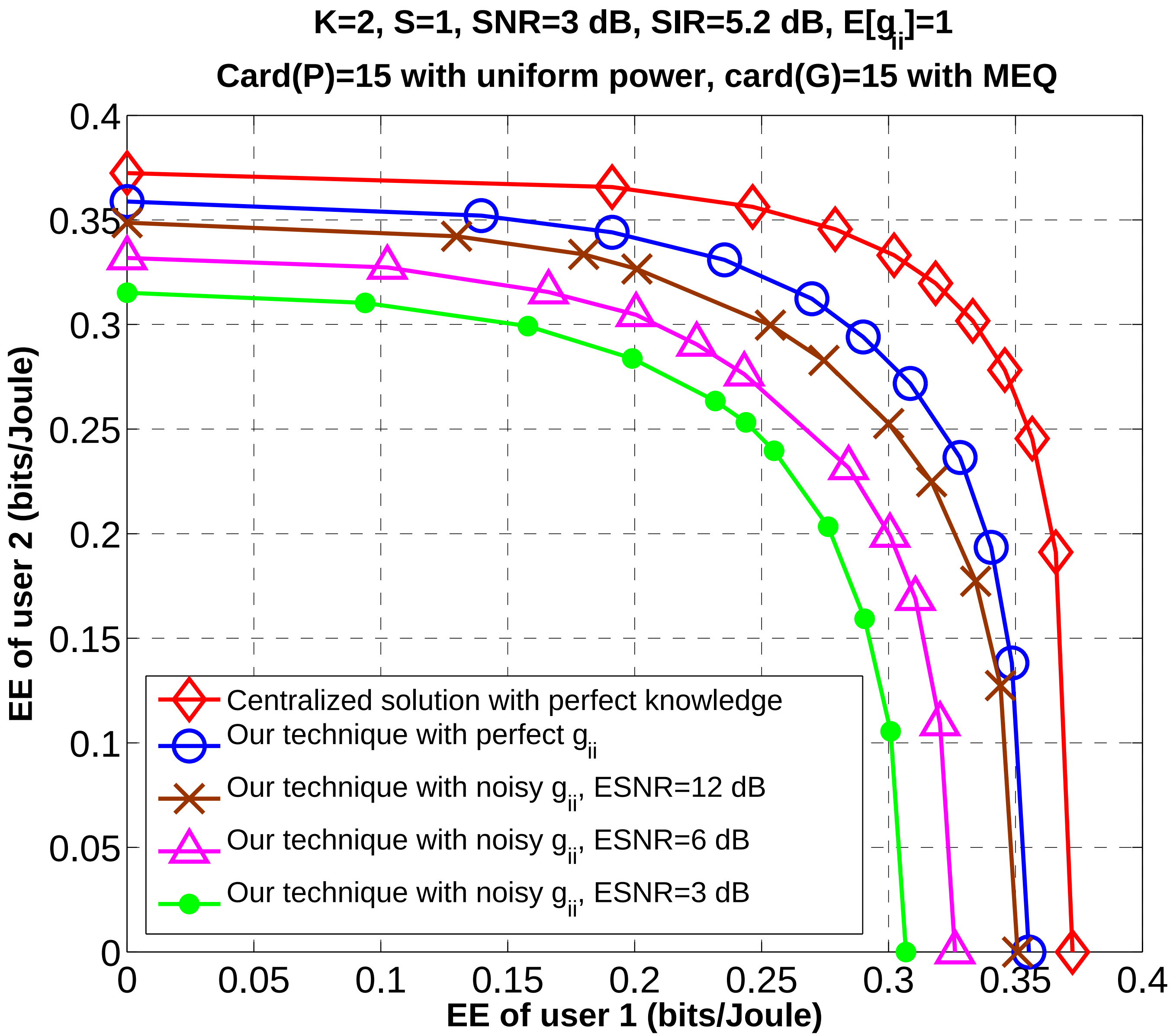
[Larrousse et al ITW 2015]

Theorem: g i.i.d., Γ_i DMC. The average utility $\overline{u} = (\overline{u}_1, ..., \overline{u}_K)$ is achievable when $T \to \infty$ iff it writes as

$$\overline{u}_i = \sum_{g,p_1,...,p_K,s_1,...,s_K,v} \rho(g)\Gamma(s_1, ..., s_K|g)P_V(v)\prod_{i=1}^{K} P_{P_i|S_i,V}(p_i|s_i, v)$$
$$\times u_i(p_1, ..., p_K; g)$$

Feasible utility region characterization

Illustration for energy-efficiency



Power control strategy (noncausal case):

$$f_{i,t} : (s_i(1), \ldots, s_i(T), y_i(1), \ldots, y_i(t-1)) \mapsto p_i(t)$$

Utility:

$$\overline{u}_i = \mathbb{E}_Q(u_i(p_1, \ldots, p_K; g))$$

Characterization for a special case

Theorem [Larrousse et al ITW 2015] $Q(g, p_1, p_2)$ implementable iff it is marginal of some

$$
\begin{aligned}
& Q(p_1, p_2, g, s_1, y_2, v) \\
& \quad = \rho_0(g) \daleth(s_1|g) P_{VP_1P_2|S_1}(v, p_1, p_2|s_1) \Gamma(y_2|g, p_1)
\end{aligned}
$$

satisfying

$$
I_Q(S_1; P_2) \leq I_Q(V; Y_2|P_2) - I_Q(V; S_1|P_2)
$$

Utility region characterization

Pareto frontier: use $w_\alpha = \alpha u_1 + (1-\alpha) u_2$

$$
\begin{array}{lrcl}
\text{minimize} & -\sum_{g,p_1,p_2} Q(g,p_1,p_2) w_\alpha(g,p_1,p_2) & & \\
\text{subject to} & H_Q(G) + H_Q(P_2) - H_Q(G,P_1,P_2) & \leq & 0 \\
 & -Q(g,p_1,p_2) & \leq & 0 \\
 & -1 + \sum_{g,p_1,p_2} Q(g,p_1,p_2) & = & 0 \\
 & -\rho_0(g) + \sum_{p_1,p_2} Q(g,p_1,p_2) & = & 0
\end{array}
$$

Take away messages

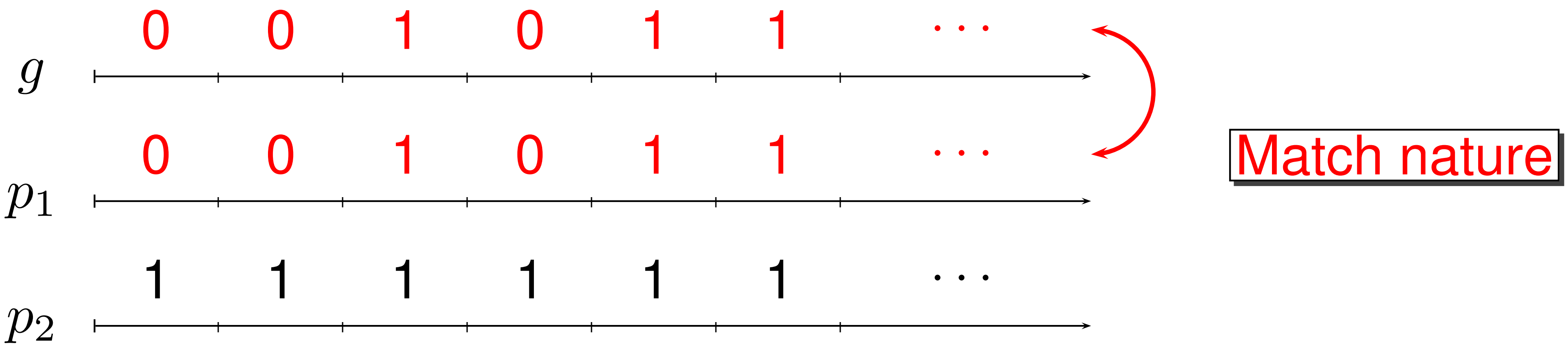
- Finding the limiting performance = solving an OP

- Open decision/game problems $\leftrightarrow$ open information theory problems

Power control code example

Long-term utility

- Scheme 1:

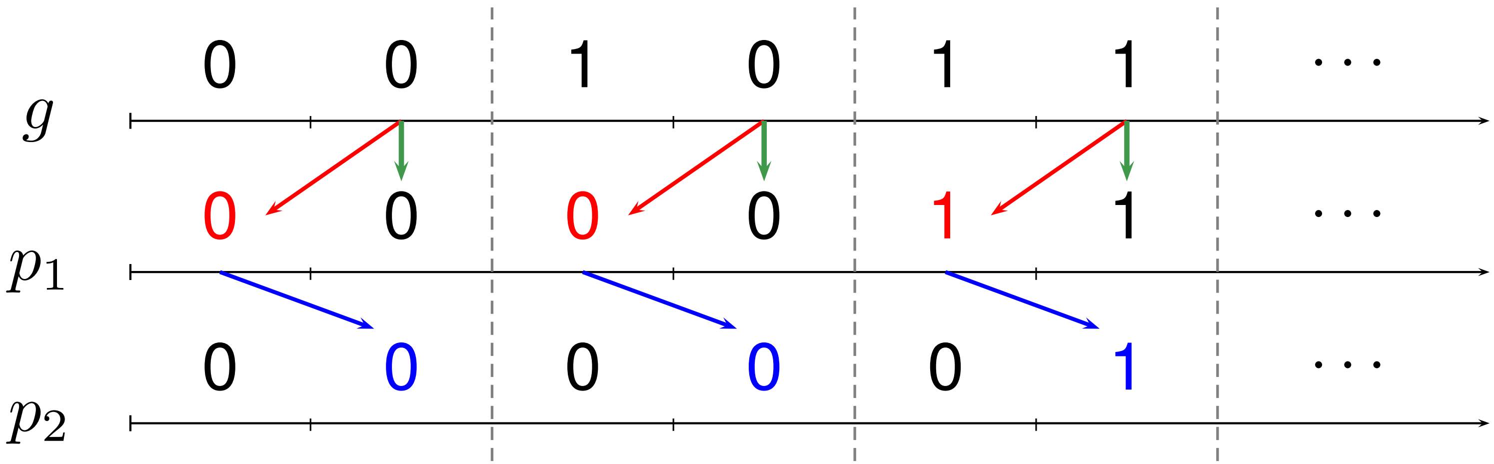


- Long-term utility

$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} u(p(t), g(t))\right] \to \frac{1}{2} = 0.5. \quad \text{for } g \sim \mathcal{B}\left(\frac{1}{2}\right)$$

Long-term utility

- Scheme 2:

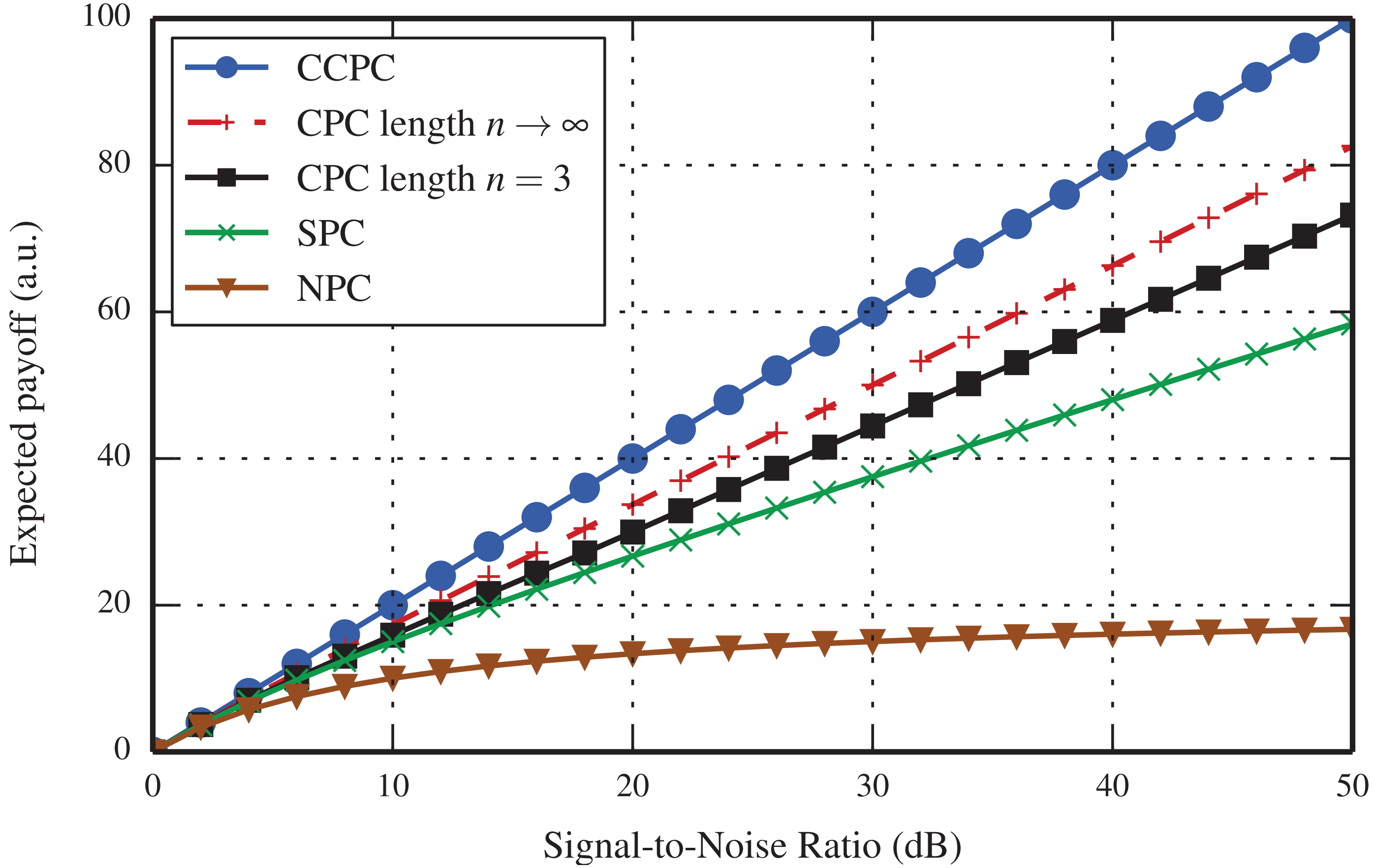


- Long term utility:

$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} u(p(t), g(t))\right] \to \frac{5}{8} = 0.625.$$

Illustration [Larrousse et al TIT 2017]

Setting Multiple access channel, $K = 2$, binary power control, BSC links, $u_i = \log(1 + \mathrm{SINR}_i)$



Technical challenges

- Construct codes (see [Larrousse and Lasaulce ISIT 2013][Larrousse et al TIT 2017]). Joint control-communication problem.

- Controlled states.

- Nash equilibrium points.

Thank you for your attention!

Main references (1/3)

Varma et al Eusipco 2015: Vineeth S. Varma, Samson Lasaulce, Chao Zhang, and R. Visoz. Power modulation: Application to inter-cell interference coordination. In EUSIPCO. IEEE, 2015.

Zhang et al 2017: C. Zhang, V. Varma, S. Lasaulce, and R. Visoz, ”Implementing Coordination in Interference Networks through Power Domain Channel Estimation”, IEEE Transactions on Wireless Communications, 2017.

Larrousse et al ITW 2015: Larrousse, B., Lasaulce, S. and Wigger, M. (2015, May). Coordination in State-Dependent Distributed Networks. In Information Theory Workshop (ITW). (pp. 1-5). IEEE.

Mertikopoulos JSAC 2012: P. Mertikopoulos, E. V. Belmega, A. Moustakas, and S. Lasaulce, ”Distributed Learning Policies for Power Allocation in Multiple Access Channels”, IEEE Journal of Selected Areas in Communications (JSAC), Vol. 30, No. 1, pp. 96–106, Jan. 2012.

Main references (2/3)

C. Zhang, N. Khalfet, S. Lasaulce, and S. Tarbouriech, ”Utility-oriented quantization and application to power control”, IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2017.

B. Larrousse, S. Lasaulce, and M. Bloch, ”Coordination in distributed networks via coded actions with application to power control”, IEEE Transactions on Information Theory. 2017.

S. Berri, S. Lasaulce, and M. S. Radjef, ”Power control with partial observation in wireless ad hoc networks”, IEEE Proc. of the EUSIPCO conference, Budapest, Hungary, Aug. 2016.

A. Agrawal, S. Lasaulce, O. Beaude, and R. Visoz, ”A framework for decentralized power control with partial channel state information”, IEEE Proc. of the Fifth International Conference on Communications and Networking (ComNet2015), Hammamet, Tunisia, November 4-7, 2015.

O. Beaude, A. Agrawal, and S. Lasaulce, ”A framework for computing power consumption scheduling functions under uncertainty”, 6th IEEE International Conference on Smart Grid Communications (SmartGridComm 2015), Miami, Florida, USA, Nov. 2015.

Main references (3/3)

A. Agrawal, F. Danard, B. Larrousse, and S. Lasaulce, ”Implicit coordination in two-agent team problems”, European Conference on Control (ECC), Linz, Austria, July 2015.

B. Larrousse, S. Lasaulce, and M. Wigger, ”Coordination in state-dependent distributed networks: The two-agent case”, IEEE International Symposium on Information Theory (ISIT), Hong Kong, June 2015.

B. Larrousse, S. Lasaulce, and M. Wigger, ”Coordination in State-Dependent Distributed Networks”, IEEE Proc. of the Information Theory Workshop (ITW), Jerusalem, Israel, Apr.-May 2015.

B. Larrousse, A. Agrawal, and S. Lasaulce, ”Implicit coordination in two-agent team problems. Application to distributed power allocation”, IEEE 12th Intl. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), Hammamet, Tunisia, May 2014.

Backup slides

The iterative water-filling algorithm (IWFA)

- At time-slot t, the water-filling solution writes as

$$p_{i,s}(t+1) = \left[\frac{1}{\lambda_i} - \frac{p_{i,s}(t)}{\mathrm{SINR}_{i,s}(t)}\right]^+$$

References: [Yu et al JSAC 2002] (multi-band); [Scutari et al TSP 2009] (MIMO)

About IWFA-type algorithms

- **Required knowledge:** $\mathrm{SINR}_{i,s}$

- **Complexity:** low

- **Convergence:** conditional [Scutari et al TSP 2009], sometimes w.p.0. [Mertikopoulos et al JSAC 2012]

About IWFA-type algorithms. Continued

► **Global efficiency:** typically not good for medium/high interference levels

$$w_{\text{sum}}(p_1, ..., p_K) = \sum_{i=1}^{K} \sum_{s=1}^{S} \log \left(1 + \text{SINR}_{i,s}\right)$$

with $p_i = (p_{i,1}, ..., p_{i,S})$

Local CSI exchange phase description

$$\underline{g}_i \xrightarrow{\text{Phase I}} \widetilde{\underline{g}}_i \xrightarrow{\text{Quant.}} Q_i^{\mathrm{II}}(\widetilde{\underline{g}}_i) \xrightarrow{\text{Mod.}} \underline{p}_i^{\mathrm{II}}$$

$$\downarrow$$

$$\widetilde{\underline{g}}_i^j \xleftarrow{\text{Dequantizer\&}\ \text{Demodulator}} \widetilde{\underline{p}}_i^{\mathrm{II}} \xleftarrow{\text{Decoder } \sharp j,\ j \neq i} \widetilde{\underline{\omega}}_j^{\mathrm{II}}$$

- What is not classical in the above operations