WIRELESS QUEUING NETWORKS UNDER CHURN AND MOTION

F. Baccelli

UT Austin & Inria Paris

Simons Center for Communication, Information & Network Mathematics UT Austin

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Structure of the Lecture

- **Background and Motivation**
- **Methodology**
- **Part 1**
  Birth and Death of Wireless Queues
  Joint work with A. Sankararaman and S. Foss
- **Part 2**
  Motion of Wireless Queues under Multihop Routing
  Joint work with S. Rybko, S. Shlosman and S. Vladimirov

Motivations in Wireless Networks

- **Lack of understanding and analysis of**

  Space-time interactions

  – **Static spatial setting well understood: Stochastic Geometry [FB, Blaszczyszyn 01]**

  – **Churn partly taken into account in flow-based queuing [Bonald, Proutiere 06], [Shakkottai, De Veciana 07]**

- **Contents of this lecture:**

  First models with such dynamics in stochastic geometry

Methodology

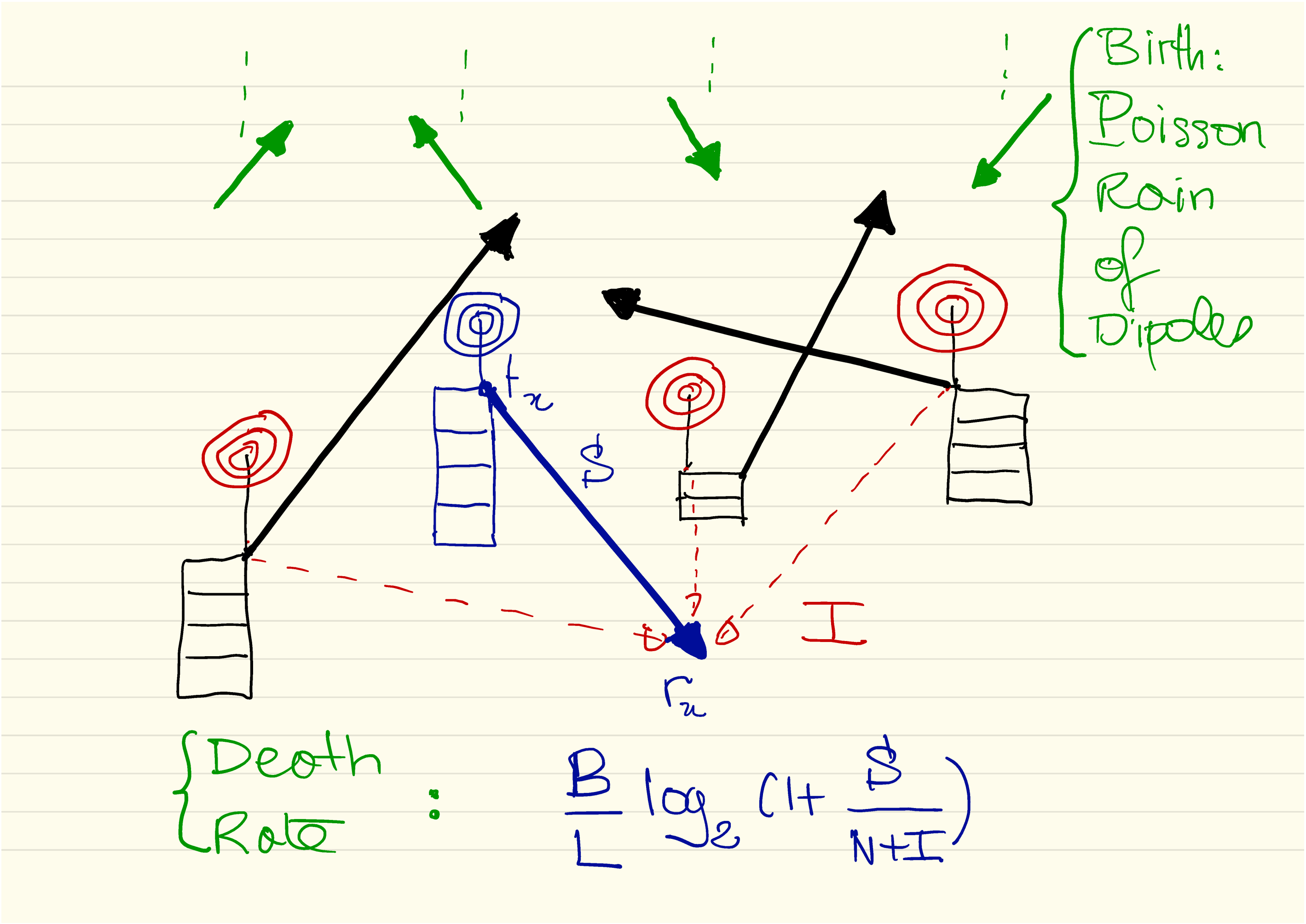
Everything Should Be Made as Simple as Possible,
But Not Simpler
A. E.

Setting

- **Infrastructureless Wireless Network**:
  Ad-hoc Networks, D2D Networks, IoT
- **Markov Models**:
  Poisson, Exponential
- **Mathematical tools**:
  Stochastic Geometry, Fluid, Mean-Field

I: Churn: Birth and Death

- **Problem Statement**
- **Summary of Results**
- **Proof Overview**


Birth: Poisson Rain of Dipoles
t_x
S
r_x
I
Death Rate :
$\frac{B}{L}\log_2(1+\frac{S}{N+I})$

Stochastic Network Model

- $\mathbf{S} = [-Q, Q] \times [-Q, Q]$: torus where the wireless links live
- **Links**: (Tx-Rx pairs)
- Links: **arrive** as a PPP on $\mathbb{R} \times \mathbf{S}$ with intensity λ:
  Prob. of a point arriving in space **dx** and time **dt**: $\lambda \mathrm{dxdt}$
- Each Tx has an **i.i.d. exponential file size** of mean **L** bits to transmit to its Rx
- A point **exits** after the Tx finishes transmitting its file
- Φ_t: set of locations of links present at time t:

$$\Phi_t = \{x_1, \ldots, x_{N_t}\}, \quad x_i \in \mathbf{S}$$

Interference and Service Rate

- **Interference seen at point x due to configuration Φ**

$$\mathbf{I}(\mathbf{x}, \boldsymbol{\Phi}) = \sum_{\mathbf{x_i} \in \boldsymbol{\Phi} \neq \mathbf{x}} \mathbf{l}(||\mathbf{x} - \mathbf{x_i}||)$$

  – **Distance on the torus**

  – $\mathbf{l}(\cdot)$**:** $\mathbb{R}^+ \to \mathbb{R}^+$**: path loss function**

- **The speed of file transfer by link at x in configuration Φ is**

$$\mathbf{R}(\mathbf{x}, \boldsymbol{\Phi}) = \mathbf{B} \log_2 \left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{N} + \mathbf{I}(\mathbf{x}, \boldsymbol{\Phi})}\right)$$

- $\mathbf{B}, \mathbf{N}$ **Positive constants**

B& D Master Equation

- **A point born at $\mathbf{x_p}$ and time $\mathbf{b_p}$ with file-size $\mathbf{L_p}$ dies at time**

$$\mathbf{d_p} = \inf \left\{ \mathbf{t} > \mathbf{b_p} : \int_{\mathbf{u}=\mathbf{b_p}}^{\mathbf{t}} \mathbf{R}(\mathbf{x_p}, \mathbf{\Phi_u}) \mathbf{du} \geq \mathbf{L_p} \right\}$$

- **Spatial Birth-Death Process**
  - **Arrivals from the Poisson Rain**
  - **Departures happen at file transfer completion**

Properties of the Dynamics

- **The statistical assumptions imply that Φ_t is a Markov Process on the set of simple counting measures on $\mathbf{S}$**
- **Euclidean extension of the flow-level models of [Bonald, Proutiere 06], [Shakkottai, De Veciana 07]**

Questions

- **Existence and uniqueness of the stationary regimes of Φ_t**
- **Characterization of the stationary regime(s) if existence**

Main Stability Results

$$\mathbf{a} := \int_{\mathbf{x}\in\mathbf{S}} \mathbf{l}(||\mathbf{x}||)\mathbf{dx}$$

- **Theorem**
  - If $\lambda > \frac{\mathbf{B}}{\ln(\mathbf{2})\mathbf{La}}$, then Φ_t admits no stationary regime.
  - If $\lambda < \frac{\mathbf{B}}{\ln(\mathbf{2})\mathbf{La}}$, and $\mathbf{r} \to \mathbf{l}(\mathbf{r})$ bounded and monotone, then Φ_t admits a unique stationary regime
- Sufficient condition by fluid limit
- **Corollary**
  For the path-loss model $\mathbf{l}(\mathbf{r}) = \mathbf{r}^{-\alpha}$, $\alpha \geq 2$, for all $\lambda > 0$, and all mean file sizes, the process Φ_t admits no stationary-regime

Main Qualitative Result

- $\mathbf{\Phi}$ **stationary point-process on S with Palm distribution** $\mathbb{P}^{\mathbf{0}}$
- **Clustering**

  Φ is clustered if for all bounded, positive, non-increasing functions $\mathbf{f}(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$, the shot noise;

$$\mathbf{F}(\mathbf{x}, \mathbf{\Phi}) := \sum_{\mathbf{y} \in \mathbf{\Phi} \setminus \{\mathbf{x}\}} \mathbf{f}(||\mathbf{y} - \mathbf{x}||)$$

  satisfies

$$\mathbb{E}^{\mathbf{0}}[\mathbf{F}(\mathbf{0}, \mathbf{\Phi})] \geq \mathbb{E}[\mathbf{F}(\mathbf{0}, \mathbf{\Phi})]$$

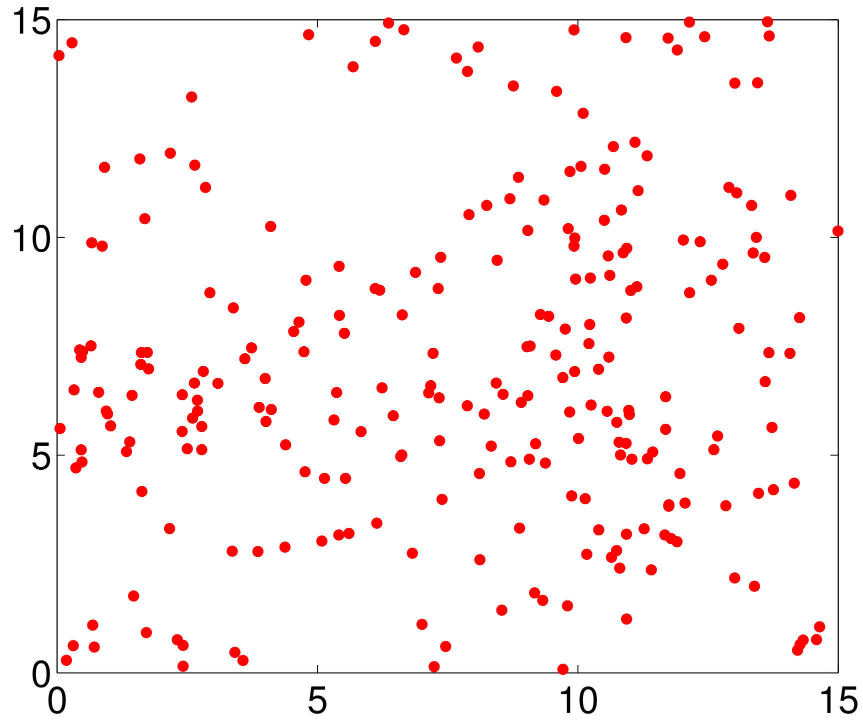
Main Qualitative Result (*continued*)

- **Theorem**
  The steady-state point process, when it exists, is clustered

- **Follows from Palm calculus + the FKG inequality**

- **Interpretation of the result**
  The steady-state interference measured at a uniformly randomly chosen point of is larger in mean than that at an uniformly random location of space.

- **Key Observation**
  - **Dynamics Shapes Geometry**
  - **Geometry Shapes Dynamics**



A sample of Φ when $\lambda = 0.99$ and $\mathbf{l}(\mathbf{r}) = (\mathbf{r} + \mathbf{1})^{-4}$.

First Quantitative Results

- **Mean-field approximations for the intensity of the steady-state process**
  1. **Poisson heuristic $\beta_{\mathbf{f}}$ - derived by neglecting clustering and assuming Poisson**
  2. **Second-order heuristic $\beta_{\mathbf{s}}$ based on a second-order cavity approximation of the dynamics**

Poisson Heuristic

Exact Rate Conservation Law:

$$\lambda \mathbf{L} = \beta \mathbb{E}^{\mathbf{0}}_{\mathbf{\Phi}} \left[\log_{\mathbf{2}} \left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{N} + \mathbf{I}(\mathbf{0})} \right) \right].$$

Poisson Heur.: Largest solution to the fixed point equation:

$$\lambda \mathbf{L} = \frac{\beta_{\mathbf{f}}}{\ln(\mathbf{2})} \int_{\mathbf{z}=\mathbf{0}}^{\infty} \frac{\mathbf{e}^{-\mathbf{N}\mathbf{z}}(\mathbf{1} - \mathbf{e}^{-\mathbf{z}})}{\mathbf{z}} \mathbf{e}^{-\beta_{\mathbf{f}} \int_{\mathbf{x} \in \mathbf{S}} (\mathbf{1} - \mathbf{e}^{-\mathbf{z}\mathbf{l}(||\mathbf{x}||)}) \mathbf{dx}} \mathbf{dz}$$

Ignores the Palm effect and uses that if $\mathbf{X}, \mathbf{Y}$ are non-negative and independent,

$$\mathbb{E} \left[\ln \left(\mathbf{1} + \frac{\mathbf{X}}{\mathbf{Y} + \mathbf{a}} \right) \right] = \int_{\mathbf{z}=\mathbf{0}}^{\infty} \frac{\mathbf{e}^{-\mathbf{a}\mathbf{z}}}{\mathbf{z}} (\mathbf{1} - \mathbb{E}[\mathbf{e}^{-\mathbf{z}\mathbf{X}}]) \mathbb{E}[\mathbf{e}^{-\mathbf{z}\mathbf{Y}}] \mathbf{dz}.$$

Second Order Heuristic

The intensity β_s is given by

$$\beta_s = \frac{\lambda L}{B \log_2 \left(1 + \frac{1}{N + I_s}\right)}$$

where I_s is the smallest solution of the fixed-point equation

$$I_s = \lambda L \int_{x \in S} \frac{l(||x||)}{B \log_2 \left(1 + \frac{1}{N + I_s + l(||x||)}\right)} dx$$

Second Order Heuristic (*continued*)

- **Rationale based on $\rho_2(\mathbf{x}, \mathbf{y})$: second moment measure of Φ**
- **Rate Conservation for ρ_2: when considering $\mathbf{I}_s$ as a constant**

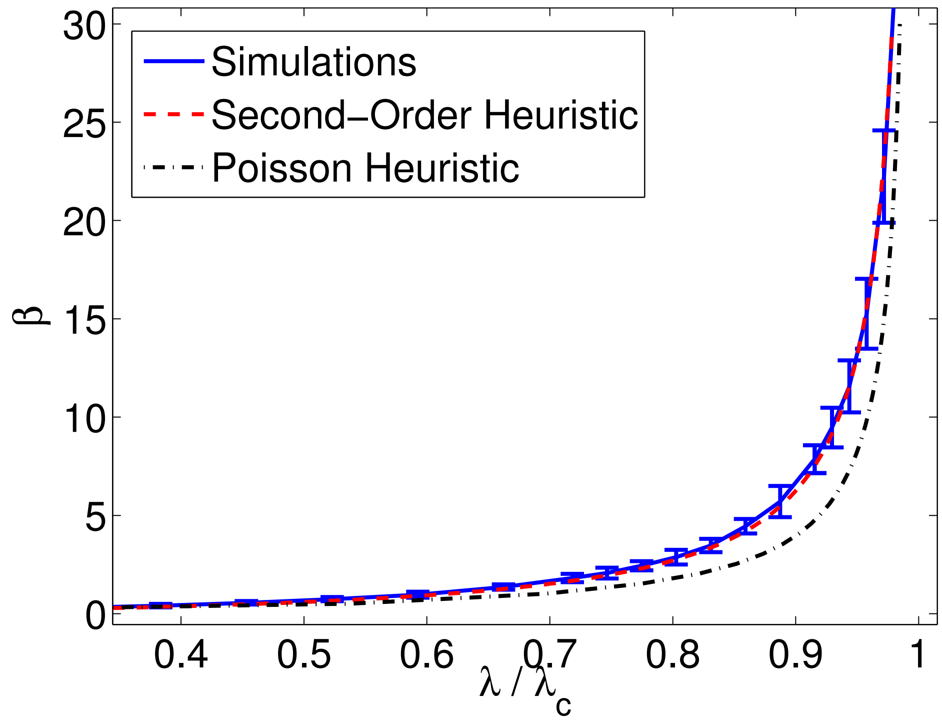
$$\rho_2(\mathbf{x}, \mathbf{y})\frac{\mathbf{1}}{\mathbf{L}}\mathbf{B}\log_2\left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{N} + \mathbf{I}_s + \mathbf{l}(||\mathbf{x} - \mathbf{y}||)}\right) = \lambda\beta_s$$

- **From the definition of second moment measure,**

$$\mathbf{I}_s = \int_{\mathbf{x} \in \mathbf{S}} \mathbf{l}(||\mathbf{x}||)\frac{\rho_2(\mathbf{0}, \mathbf{x})}{\beta_s}\mathbf{dx}$$

which gives the fixed point equation for $\mathbf{I}_s$

- **The formula for β_s follows from Rate Conservation for $\rho_1 = \beta_s$**



95% **confidence interval when** $\mathbf{l(r) = (r+1)^{-4}}$

Tightness Results & Extensions

- The **Poisson heuristic** is **tight** in **heavy and light traffic**
- **Recent Extensions** obtained with **S. Foss**:
  - **Exact expression** for the intensity β of Φ in the **Low SINR** regime when replacing the death rate by

$$\frac{\mathbf{B}}{\ln(\mathbf{2})\mathbf{L}} \frac{\mathbf{S}}{\mathbf{N}+\mathbf{I}(\mathbf{x},\boldsymbol{\Phi})}$$

  - **Scalability result**: extension to dynamics on $\mathbb{R}^2$ using **Coupling from the Past** techniques.
- **Future**: introduction of **scheduling or multi-user IT**

Summary

- A new basic representation of space time interactions in wireless networks
- A generative model for clustering as assumed in 3GPP simulation standards
- A new dynamic notion of capacity involving both queuing and IT
- First analytical results in the Low SINR case and good heuristics in general

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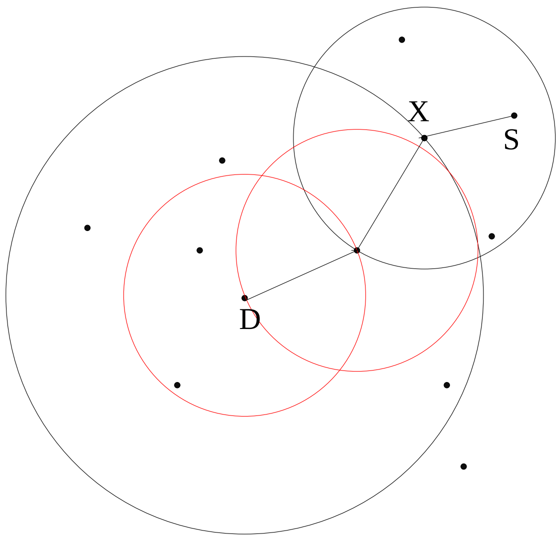
II: Motion and Multi-Hop Routing

- **Problem Statement**
- **Summary of Results**

Problem Statement

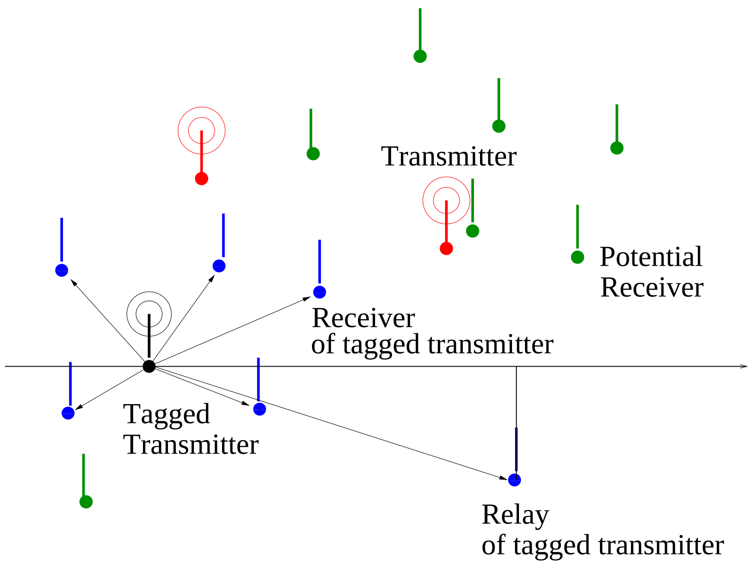
- **Setting: Grossglauser & Tse 02 scaling law problem**
  - **Multihop relaying**
  - **Opportunistic geographic routing**
  - **Motion of nodes**
- **New SG+QT view of the problem**

Example of Geographic Routing



- **Nearest Neighbor Geographic Routing** The next hop on the route from S to D is the nearest among the nodes which are closer from D than X.
- **On a Poisson P.P., a.s.**
  - **No ties**
  - **Converges in finite number of steps**

Wireless Geographic Routing



- Each node uses Aloha to split the Poisson p.p. into transmitters and potential receivers
- Potential relays of a transmitters: receivers with a large enough SINR
- Geographic Routing: next hop:= potential relay nearest to destination

Traffic and Relaying

- **DTN type assumptions:**
  - **Wireless nodes move**
  - **Each moving node generates packets**
  - **Each generated packet has a destination, e.g. another node at finite distance**
  - **Multihop relaying: each node transmits**
    - **its own packets**
    - **those of other nodes on their route to destination**
  - **Contention: on each node, packets are queued FIFO and are served as above (SINR condition)**

Reduction (as per Methodology :)

- **Wireless nodes move randomly on a grid or a graph G (e.g. $\mathbb{Z}$ or $\mathbb{Z}/\mathbf{K}\mathbb{Z}$, $\mathbb{Z}^2$, d-regular graph)**
- **Traffic:**
  - **Each moving node generates packets at rate λ**
  - **Each generated packet has a destination (e.g. a point of the grid, vertex of the graph)**
- **Contention: on each node packets are queued FIFO**

Reduction (as per Methodology :) (*continued*)

- **Motion**: neighbor nodes swap their positions with rate β
- **Wireless**: communication to neighbors only
- **Opportunistic multihop routing**:
  upon service at a node, a packet:
  – is routed to the neighbor the closest to its destination
  – leaves the system if the destination is distance 1 or 0

Finite Network Markovization

- **Assumptions**
  - **Poisson arrivals with intensity λ**
  - **exponential service times with mean 1**
  - **finite connected graph with K nodes**
- **Markov representation with discrete non compact state:**
  - **Permutation on $[1, \ldots, K]$**
    (locations of wireless nodes)
  - **Ordered queue at each node**
    (finite ordered list of destinations)

Finite Network Instability Result

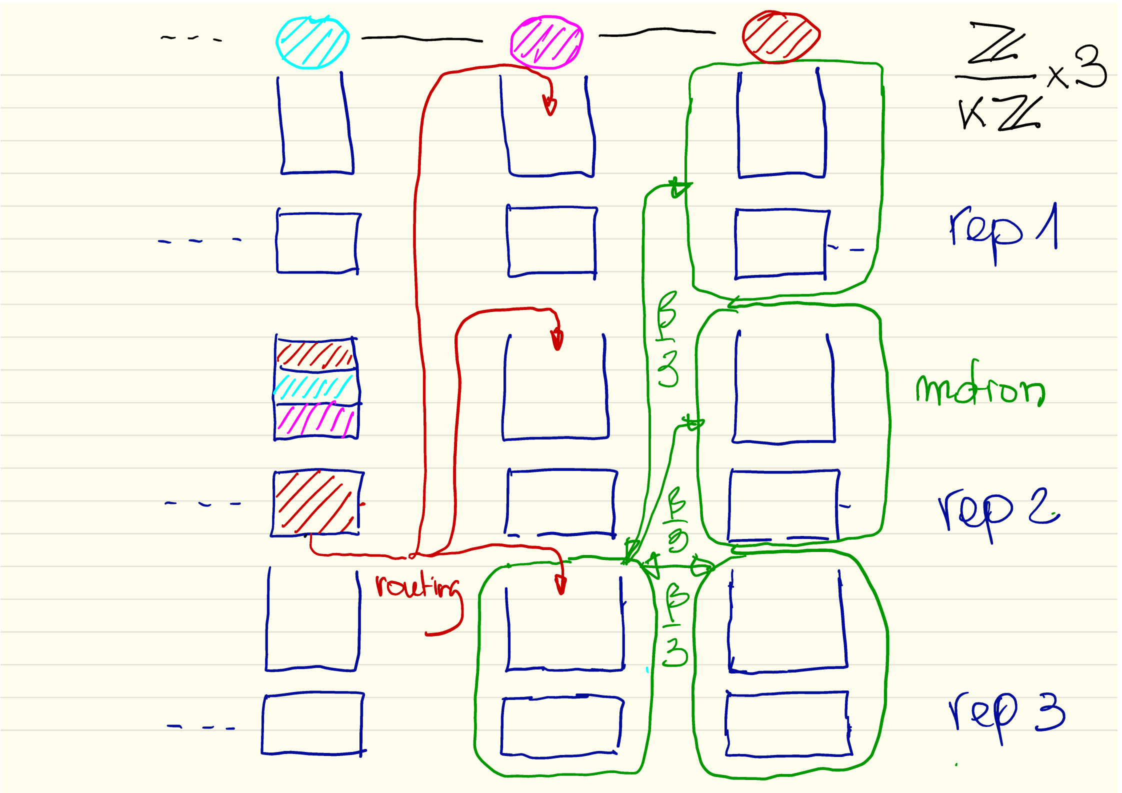
- **Maximal degree in the graph: $\mathbf{d}$**
- **Motion rate β**
- **Theorem**
  For all $\beta > 0$, this Markov process is transient when
  $$\mathbf{K} > \mathbf{K}^* = \frac{\mathbf{d}+1}{\lambda}$$
- **For all load factors, a finite network is unstable when the diameter of the network is large enough!**

Finite Network Instability Result (*continued*)

- **Sketch of Proof in the d-regular case**
  - **nodes mix to the uniform distribution on $[1, \ldots, K]$**
  - **the proportion of time the server harboring a packet is a neighbor of its destination vertex is order $\frac{d+1}{K}$.**
  - **if $\lambda > \frac{d+1}{K}$, drift inside orthant is positive on all coordinates**
  - **supermartingale argument**

Replicated Version

- **N-Replica version of the network on G**
- **New graph G_N with**
  - $V_N = V \times \{1, \ldots, N\}$
  - **edge between (u, i) and (v, j) if (u, v) edge in G**
- **Routing**
  - **destination is any replica of the destination vertex in G**
  - **routing to one of the N replicas closest to destination at random**
- **Swaping: between a replica and any neighboring replica at random**



The 3-Replica Version of $\mathbb{Z}/\mathbf{K}\mathbb{Z}$

Instability of the N-Replica Version of a Finite Network

- **Same instability result for $\mathbf{G_N}$ as for G for fixed N**
- **Example The N-replica version of the network on $\frac{\mathbb{Z}}{\mathbf{K}\mathbb{Z}}$ is unstable under the same condition as the network on $\frac{\mathbb{Z}}{\mathbf{K}\mathbb{Z}}$ namely if $\mathbf{K > 3/\lambda}$**

Mean-Field Version

- **Mean-Field version of the network on G:**
  weak limit of the latter when N tends to infinity.
- **Notation: network on G^{∞}**
- **Existence: tightness arguments**

Mean Field Networks on $\mathbb{Z}$

- **Non Linear Markov Process roughly, dynamical system on probability measures μ on queue states**

$$\mu(\mathbf{q}) = \mu(\mathbf{n_1}, \ldots, \mathbf{n_l}), \quad \mathbf{n_k} : \textbf{relative location of dest}(\mathbf{c_k})$$

- **Functional equation for fixed points $\mu(\mathbf{q})$ of this dynamical system**

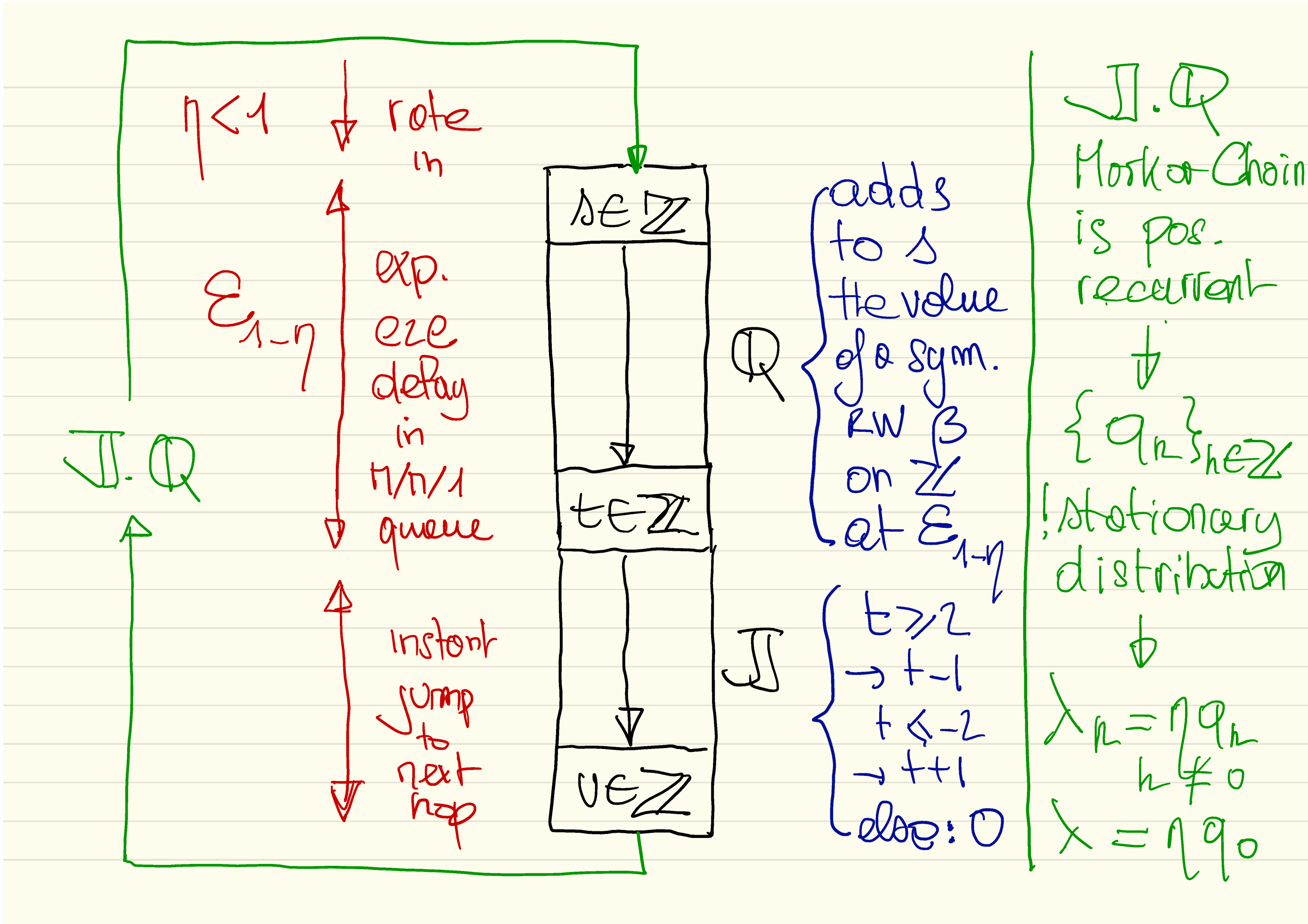
Mean Field Networks on $\mathbb{Z}$ (*continued*)

- **Theorem For all $0 < \eta < 1$, there exists**
  a unique $0 < \lambda = \lambda_\eta < 1$
  a unique probability distribution $\mu = \mu_\eta$ on the queue state
  such that, for the exogenous arrival λ,

  – **μ is solution of the functional equation and**

  – **the total rate in a node under μ is η**

- **Sketch of Proof**
  Special case where the destination is the vertex of birth


η<1
rate in
ε1-η
exp. rate delay in M/M/1 queue
instant jump to next hop
J.Q
s∈ℤ
t∈ℤ
v∈ℤ
Q
adds to s the value of a sym. RW β on ℤ at ε1-η
J
t≥2 → t-1
t≤-2 → t+1
else: 0
J.Q
Markov Chain is pos. recurrent
{qn}n∈ℤ
! stationary distribution
λn = η qn
n ≠ 0
λ = η q0

Existence of Multiple Solutions

- **Theorem** **For the mean-field version of the network on $\mathbb{Z}$, there exists a λ_* such that for all $\lambda < \lambda_*$, there are at least two different values $\eta = \eta_-(\lambda)$ and $\eta = \eta_+(\lambda)$ s.t.**

  – $\lambda(\eta) = \lambda$

  – $\eta_-(\lambda) \rightarrow \mathbf{0}$ **as** $\lambda \rightarrow \mathbf{0}$

  – $\eta_+(\lambda) \rightarrow \mathbf{1}$ **as** $\lambda \rightarrow \mathbf{0}$

- **Sketch of Proof**

  – **When** η **tends to 0,** $\lambda(\eta) = \lambda \mathbf{q_0}$ **tends to 0 by M/M/1**

  – **When** η **tends to 1,** $\lambda(\eta) = \lambda \mathbf{q_0}$ **tends to 0 by M/M/1 as well**

Illustration

- **MC of the distance to destination:**

$$\mathbf{p(n, n+1)} = \beta, \quad \mathbf{p(n, n-1)} = \beta + \gamma,$$

with $\gamma = 1 - \eta$**,**

$$\mathbf{p(1, 2)} = \beta, \quad \mathbf{p(1, 0)} = \beta, \quad \mathbf{p(1, *)} = \gamma$$

and

$$\mathbf{p(0, 1)} = \mathbf{2}\beta, \quad \mathbf{p(0, *)} = \gamma,$$

where $*$ **is absorbing**

Illustration (*continued*)

- **Mean # nodes visited by a customer till absorption:**

$$\mathbb{E}[\mathbf{N}] = \mathbf{1} + \frac{\mathbf{2}\beta^{\mathbf{2}}}{\gamma(\mathbf{3}\beta + \gamma)}$$

- **Flow equation:**

$$\eta = \mathbf{1} - \gamma = \lambda\left(\mathbf{1} + \frac{\mathbf{2}\beta^{\mathbf{2}}}{\gamma(\mathbf{3}\beta + \gamma)}\right)$$

When β is large, there are two roots

$$\eta^{-} = \lambda + \frac{\mathbf{1} - \lambda - \sqrt{(\mathbf{1} - \lambda)^{\mathbf{2}} - \frac{\mathbf{8}}{\mathbf{3}}\lambda\beta}}{\mathbf{2}}$$

$$\eta^{+} = \lambda + \frac{\mathbf{1} - \lambda + \sqrt{(\mathbf{1} - \lambda)^{\mathbf{2}} - \frac{\mathbf{8}}{\mathbf{3}}\lambda\beta}}{\mathbf{2}}$$

with

$$0 < \eta^{-} < \eta^{+} < 1, \qquad \nu^{-} \to 0, \quad \nu^{+} \to 1, \quad \textbf{when } \lambda \to 0$$

Generalization

- **The multiple fixed point result can be extended to the Cayley graph of any group G s.t.**
  - **G has a finite generating set**
  - **G is infinite**
  - **the arrival rate, swap rate, swap rule, destination rule are G-invariant**
- **Example: the network of $(\mathbb{Z})^\infty$ has at least to fixed points stationary regimes for every $\lambda < \lambda_*$ with $\lambda_* > 0$.**

Meta-Stability

- **Finitely many replicas–Infinitely many replicas stability difference.**
- **No contradiction with the fact that, for $\mathrm{N} < \infty$, the network of $(\mathbb{Z})^{\mathrm{N}}$ has no stationary regimes**
  the time - replica diagram does not commute here!

Current and Future Extensions

- **Wireless primitives**: (SINR, MAC) to assess transmission between nearby nodes
- **Motion primitives**: Brownian, random waypoint
- **Queuing primitives**: beyond Poisson arrivals and exponential service times
- **Scheduling**: beyond FIFO: needed to see why motion increases capacity

Summary

- **Way to go** **but first step of a mean-field representation of** **mutihop routing** **in wireless networks beyond scaling laws**
- **Meta-stability** **is encouraging news**
- **When fully interconnected with SG, new** **analytical handle** **for optimization in this class of problems.**

References

- Part 1
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    Infinite spatial birth-death wireless networks,
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- Part 2
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